Part Two

# DIAGNOSTIC TEST

# AP CALCULUS AB AND BC DIAGNOSTIC TESTS

Take a moment to gauge your readiness for the AP Calculus exam by taking either the AB diagnostic test or the BC diagnostic test, depending on which test you plan to take. The questions in this diagnostic are designed to cover most of the topics you will encounter on the AP Calculus exam. After you take it, you can use the results to give yourself a general idea of the subjects in which you are strong and the topics you need to review more. You can use this information to tailor your approach to the following review chapters. Ideally you'll still have time to read all the chapters, but if you're pressed for time, you can start with the chapters and subjects you really need to work on.

Important note for students planning to take the BC exam: The BC exam consists of both AB and BC material. If you plan to take the BC exam, you should test yourself on **BOTH** diagnostic tests. The BC diagnostic focuses on BC material in an effort to give you the greatest opportunity to test your understanding of BC topics. The real BC exam will not emphasize BC topics as heavily, so you should take the AB diagnostic as well to ensure that you have a good grasp of *both* the AB and the BC material.

Give yourself 30 minutes for the 20 multiple-choice questions and 20 minutes for the free-response question. Time yourself, and take the entire test without interruption—you can always call your friend back *after* you finish. Also, no TV or music. You won't have either luxury while taking the real AP Calculus exam, so you may as well get used to it now.

Be sure to read the explanations for all questions, even those you answered correctly. Even if you got the problem right, reading another person's answer can give you insights that will prove helpful on the real exam.

Good luck on the diagnostic!

# Diagnostic Test AB Answer Grid

To compute your score for this diagnostic test, calculate the number of questions you got right, then divide by 20 to get the percentage of questions that you answered correctly.

The approximate score range is as follows:

**5** = 80–100% (extremely well qualified)

**4** = 60–79% (well qualified)

**3** = 50–59% (qualified)

**2** = 40–49% (possibly qualified)

**1** = 0–39% (no recommendation)

A score of 49% is a 2, so you can definitely do better. If your score is low, keep on studying to improve your chances of getting credit for the AP Calculus AB exam.

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# DIAGNOSTIC TEST AB

**Directions:** Solve the following problems, using available space for scratchwork. After examining the form of the choices, decide which one is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

**Note:** For the actual test, no scrap paper is provided.

**In this test:**

(1) The domain of a function $f$ is the set of all real numbers $x$ for which $f(x)$ is a real number, unless otherwise specified.

(2) The inverse of a trigonometric function $f$ may be indicated using the inverse function notation $f^{-1}$ or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

1. Evaluate $\lim\limits_{h \to \frac{1}{2}} \dfrac{e^h - \sqrt{e}}{h - \frac{1}{2}}$.

   (A) $e^2$
   (B) $e^2 - 1$
   (C) $\sqrt{e}$
   (D) $e$
   (E) The limit does not exist.

2. $\int_0^{\frac{\pi}{2}} \sqrt{\sin x}\cos x \ dx =$

   (A) 0
   (B) 1
   (C) $\sqrt{\pi}$
   (D) $\dfrac{2}{3}$
   (E) $\dfrac{2\pi}{3}$

3. The graph of $y = x^3 + 21x^2 - x + 1$ is concave down for

   (A) $x < -7$
   (B) $-7 < x < 7$
   (C) all $x$
   (D) $x > 7$
   (E) $x < -7$ and $x > 7$

4. If $F(x) = \dfrac{x^3 + x^5}{\ln(x^2)}$, then $F'(\sqrt{e}) =$

   (A) $e^2(3e+1)$
   (B) $e(3e+1)$
   (C) $2e$
   (D) $\sqrt{e}(e+1)$
   (E) $3e^2 - 1$

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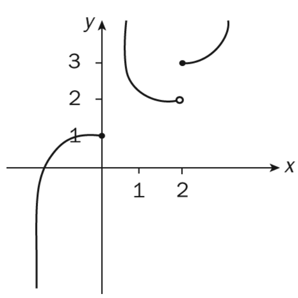
5. What are the values for which the function $f(x) = \frac{2}{3}x^3 - x^2 - 4x + 3$ is increasing?
   (A) $-1 < x < 2$
   (B) $x < -1$
   (C) $x < -1$ and $x > 2$
   (D) $0 < x < 2$
   (E) $x > -1$

6. If $f(x) = \cos^2(\sin(2x))$, then $f'\left(\frac{\pi}{8}\right) =$
   (A) $1 + \sqrt{2}\cos\left(\frac{\sqrt{2}}{2}\right)$
   (B) $\sqrt{2}\sin\left(\frac{\sqrt{2}}{2}\right)$
   (C) $\cos\left(\sin\left(\frac{\sqrt{2}}{2}\right)\right)$
   (D) $-2\sqrt{2}\cos\left(\frac{\sqrt{2}}{2}\right)\sin\left(\frac{\sqrt{2}}{2}\right)$
   (E) $-\sqrt{2}\cos\left(\frac{\sqrt{2}}{2}\right)$

7. The graph of $f$ is given below. Which of the following statements is true about $f$?




   (A) $\lim_{x \to 0} f(x) = 1$
   (B) $\lim_{x \to 0^-} f(x) = 2$
   (C) $\lim_{x \to 2^+} f(x) = 3$
   (D) $\lim_{x \to 2} f(x) = 2$
   (E) $\lim_{x \to 2^-} f(x) = \text{undefined}$

8. Evaluate $\lim_{x \to 0} \frac{\sin^2(4x)}{x^2}$.
   (A) $\frac{1}{4}$
   (B) 0
   (C) 16
   (D) 4
   (E) The limit does not exist.

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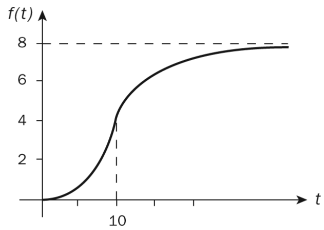
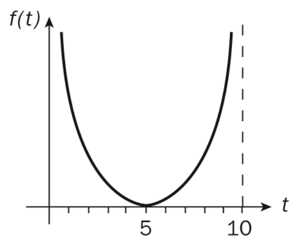
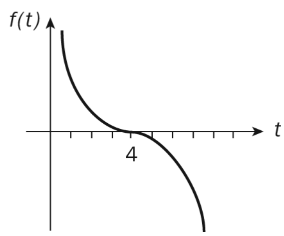
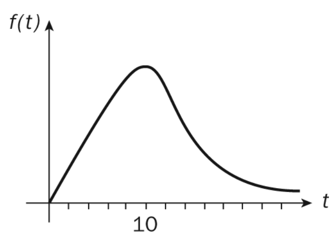
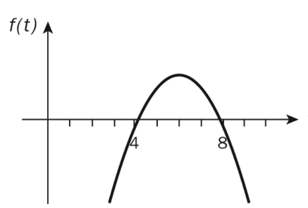


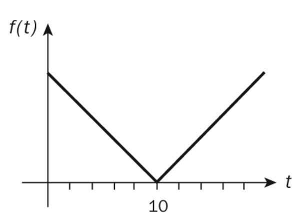

9. The graph of $f$ is given above. Which graph below could represent the graph of $f'$?

(A) 


(B) 


(C) 


(D) 


(E) 


10. Evaluate $\lim_{x\to\infty} \frac{x^3+3x^5+x}{2x^5+3x^2+5}$.

(A) 0
(B) $-\frac{4}{3}$
(C) $\frac{2}{3}$
(D) $\frac{3}{2}$
(E) The limit does not exist.

11. Evaluate $\lim_{x\to 1} \frac{2x^2+x-3}{1-x^2}$.

(A) $-\frac{5}{2}$
(B) 0
(C) $\frac{4}{3}$
(D) $-\frac{3}{2}$
(E) The limit does not exist.

12. The solution to the differential equation $\frac{dy}{dx}=\frac{x^2}{y^3}$, where $y(3)=3$ is

(A) $y=\sqrt[4]{\frac{3}{4}x^3-45}$
(B) $y=\sqrt[4]{\frac{4}{3}x^3+45}$
(C) $y=\sqrt[4]{\frac{4}{3}x^4+5}$
(D) $y=\sqrt[4]{\frac{4}{3}x^4}+\sqrt[4]{45}$
(E) $y=\sqrt[4]{\frac{4}{3}x^4+45}$

13. The slope of the tangent to the curve $2x^3y^2-5x^2y=18$ at the point $(1,2)$ is

(A) $-\frac{4}{5}$
(B) $-\frac{3}{2}$
(C) 0
(D) $-\frac{4}{3}$
(E) $-1$

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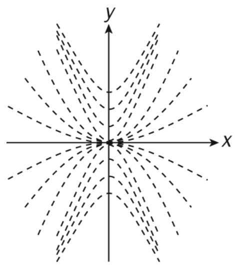
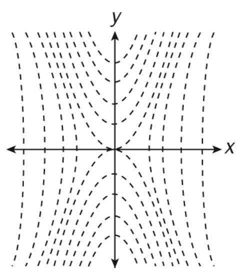
14. Which of the following is a slope field for the differential equation $\frac{dy}{dx} = \frac{2x^3}{y}$?

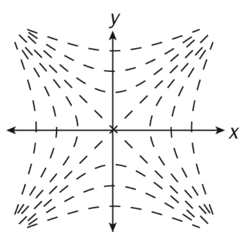
(A)



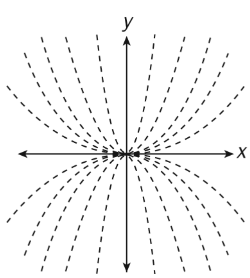
(B)



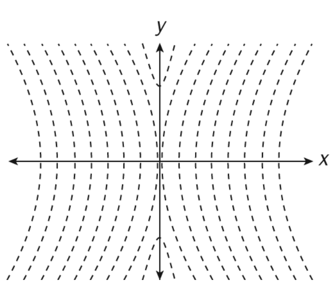
(C)



(D)



(E)



15. The area of the region enclosed by the graph of $y = 2x^2 + 1$ and the line $y = 2x + 5$ is

(A) 5
(B) 3
(C) $\frac{7}{2}$
(D) 9
(E) 1

16. At what point on the graph of $y = 2x^{\frac{3}{2}}$ is the tangent line perpendicular to the line $2x + 3y = 6$?

(A) (0, 0)
(B) $\left(\frac{1}{16}, \frac{1}{32}\right)$
(C) $\left(\frac{3}{2}, \frac{3\sqrt{6}}{2}\right)$
(D) $\left(\frac{1}{4}, \frac{1}{4}\right)$
(E) $\left(\frac{3}{16}, \frac{3\sqrt{3}}{32}\right)$

17. If the region enclosed by the $x$-axis, the line $x = 1$, the line $x = 3$, and the curve $y = x^{\frac{3}{2}}$ is revolved around the $x$-axis, the volume of the solid is

(A) $20\pi$
(B) $\frac{25}{2}\pi$
(C) $21\pi$
(D) $4\omega$
(E) $\frac{100}{3}\pi$

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18. An equation of the line tangent to the graph of $y = 2\sin\left(2x + \frac{3\pi}{4}\right)$ at $x = \frac{\pi}{8}$ is

    (A) $y - \frac{\pi}{2} = -4x$

    (B) $y + \frac{\pi}{4} = \frac{3}{4}x$

    (C) $y = 4x + \frac{\pi}{2}$

    (D) $y + \frac{\pi}{2} = 4x$

    (E) $2y = -4x + \pi$

19. Find the derivative of $f(x) = \int_0^{x^2} \cos(t^2)\,dt$.

    (A) $\cos(x^4)$

    (B) $\cos(x^2)$

    (C) $2x\cos(x^2)$

    (D) $x\cos(x^4)$

    (E) $2x\cos(x^4)$

20. Suppose that $f$ is a continuous function and differentiable everywhere. Suppose also that $f(0) = 1, f(5) = -4, f(-5) = -3$. Which of the following statements must be true about $f$?

    I. $f$ has exactly two zeros.

    II. $f$ has at least two zeros.

    III. $f$ must have a zero between 0 and $-5$.

    IV. There is not enough information to determine anything about the zeros of $f$.

    (A) I only

    (B) II only

    (C) I and III only

    (D) II and III only

    (E) IV

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## FREE-RESPONSE QUESTION

**Directions:** Solve the following problem, using available space for scratchwork. Show how you arrived at your answer.

- You should write out all your work for each part. On the actual test, you will do this in the space provided in the test booklet. Be sure to write clearly and legibly. If you make a mistake, you can save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.
- Show all your work. Clearly label any functions, graphs, tables, or other objects that you use. On the actual exam, you will be graded on the correctness and completeness of your methods as well as your answers. Answers without any supporting work may not receive credit.
- Justifications (i.e., the request that you "justify your answer") require that you give mathematical (non-calculator) reasons.
- Work must be expressed in standard mathematical notation, not calculator syntax.
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified.
- If you use decimal approximations in calculations, the readers of the actual exam will grade you on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of function $f$ is the set of all real numbers $x$ for which $f(x)$ is a real number.

21. A particle moves along the $x$-axis so that its velocity at any time $t > 0$ is given by $v(t) = 5t^2 - 4t + 7$. The position of the particle, $x(t)$, is 8 for $t = 3$.
    (a) Write a polynomial for the position of the particle at any time $t \geq 0$.
    (b) Find the total distance traveled by the particle from time $t = 0$ until time $t = 2$.
    (c) Does the particle achieve a minimum velocity? And if so what is the position of the particle at this time?

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IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS SECTION ONLY. DO NOT TURN TO ANY OTHER SECTION IN THE TEST.

STOP

## ANSWERS AND EXPLANATIONS

1. C

This problem is a direct application of the definition of the derivative. Remember: $f'(a) = \lim_{h \to a} \frac{f(h) - f(a)}{h - a}$.

Here $f(x) = e^x$ and $a = \frac{1}{2}$. We know that $f'\left(\frac{1}{2}\right) = \frac{d}{dx}e^x\Big|_{x=\frac{1}{2}} = e^{\frac{1}{2}} = \sqrt{e}$ and $f'\left(\frac{1}{2}\right) = \lim_{h \to 0} \frac{e^h - \sqrt{e}}{h - \frac{1}{2}}$. So the result follows.

2. D

Let $u = \sin x$; $du = \cos x\, dx$. We can rewrite the integral as:

$$\int_0^{\frac{\pi}{2}} \sqrt{\sin x} \cos x\, dx = \int u^{\frac{1}{2}} du = \frac{2}{3}u^{\frac{3}{2}} = \frac{2}{3}(\sin x)^{\frac{3}{2}}\Big|_0^{\frac{\pi}{2}} = \frac{2}{3}(1)^{\frac{3}{2}} - \frac{2}{3}(0)^{\frac{3}{2}} = \frac{2}{3}.$$

3. A

The function $f$ is concave down for all $x$ such that $f''(x) < 0$. By direct calculation, $y' = 3x^2 + 42x - 1$ and $y'' = 6x + 42$. So $f$ is concave down when $6x + 42 < 0$, that is, when $x < -7$.

4. B

To calculate the derivative of a quotient we must use the quotient rule:

$\left[\frac{f}{g}\right]' = \frac{gf' - fg'}{g^2}$. With $f(x) = x^3 + x^5$ and $g(x) = \ln(x^2)$,

we get $F'(x) = \frac{[\ln(x^2)][3x^2 + 5x^4] - [x^3 + x^5][\frac{1}{x^2} \cdot 2x]}{[\ln(x^2)]^2}$.

And since $\ln(e) = 1$, we get, after much reducing, $F'(\sqrt{e}) = e(3e + 1)$.

5. C

The function $f$ is increasing for all $x$ such that $f'(x) > 0$. By direct calculation, $f'(x) = 2x^2 - 2x - 4 = 2(x^2 - x - 2) = 2(x - 2)(x + 1)$. Therefore, $f'(x) > 0$ when $(x - 2)$ and $(x + 1)$ are either both positive or both negative. This occurs when $x < -1$ and $x > 2$.

6. D

In this problem we have to use the chain rule three times. Perhaps it helps to rewrite $f$ as $f(x) = [\cos(\sin(2x))]^2$. Then by applying the chain rule (three times) we get $f'(x) = 2[\cos(\sin(2x))] \times [-\sin(\sin(2x))] \times \cos(2x) \times 2 = -4\cos(\sin(2x))\sin(\sin(2x))\cos(2x)$. Now by plugging in the value $x = \frac{\pi}{8}$ and using the fact that $\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$, we get $f'\left(\frac{\pi}{8}\right) = -2\sqrt{2}\cos\left(\frac{\sqrt{2}}{2}\right)\sin\left(\frac{\sqrt{2}}{2}\right)$.

7. C

We can easily calculate each limit and check which graphs are plausible. Choice (C) is correct because as $x$ approaches 2 from the right, $f(x)$ approaches 3. The limit in (A) is undefined because the left and right side limits are not equal. For (B), by examining the graph, the limit of $f$ as $x$ approaches 0 from the left is 1, not 2. For (D), the left hand and right hand limits are not equal so the limit does not exist. For (E), the left hand limit of the function is defined. In fact, we get $\lim\limits_{x\to 2^-} f(x) = 2$.

8. C

Students are expected to know that $\lim\limits_{x\to 0}\frac{\sin x}{x} = 1$. For this problem, the limit can be rewritten:

$\lim\limits_{x\to 0}\frac{\sin^2(4x)}{x^2} = \lim\limits_{x\to 0}\left(\frac{\sin 4x}{x}\right)^2 = \lim\limits_{x\to 0}\left(4 \times \frac{\sin 4x}{4x}\right)^2 = 16 \times \lim\limits_{x\to 0}\left(\frac{\sin 4x}{4x}\right)^2 = 16$. Note that as $x \to 0, 4x \to 0$, so the limit is still 1.

9. C

Notice that the graph of $f$ has a point of inflection at $x = 10$; i.e., it changes concavity at $x = 10$. Points of inflection can only occur when $f'' = 0$ or is undefined. Since $f''$ is the rate of change of $f'$, the slope of the tangent line to the graph of $f'$ must be 0 at the point of inflection. We can therefore eliminate all but (C).

10. D

Because the degree of the numerator polynomial is equal to the degree of the denominator polynomial, the limit as $x$ approaches infinity is simply the ratio of the coefficients of the highest-order terms. Note that

$$\lim_{x\to\infty}\frac{x^3 + 3x^5 + x}{2x^5 + 3x^2 + 5} = \lim_{x\to\infty}\frac{\frac{x^3}{x^5} + \frac{3x^5}{x^5} + \frac{x}{x^5}}{\frac{2x^5}{x^5} + \frac{3x^2}{x^5} + \frac{5}{x^5}} = \frac{0 + 3 + 0}{2 + 0 + 0} = \frac{3}{2}.$$

11. A

The limit is indeterminate because if we plug in the value $x = 1$ we get $\frac{0}{0}$. However, by factoring the numerator and the denominator we can reduce the argument and easily plug in the value $x = 1$ to evaluate the limit. See that $\lim\limits_{x\to 1}\frac{2x^2 + x - 3}{1 - x^2} = \lim\limits_{x\to 1}\frac{(2x + 3)(x - 1)}{(1 - x)(1 + x)} = \lim\limits_{x\to 1}\frac{-(2x + 3)}{1 + x} = \frac{-5}{2}$.

12. B

First, cross-multiply and integrate both sides to get $\int y^3 dy = \int x^2 dx$. Thus $\frac{y^4}{4} = \frac{x^3}{3} + C$ and so, solving for $y$, we get $y = \sqrt[4]{\frac{4}{3}x^3 + C}$. Now using the initial condition we can figure out what $C$ should be. We know that $3 = \sqrt[4]{\frac{4}{3}3^3 + C}$; solving for $C$ we get $C = 45$. Therefore, we can conclude that $y = \sqrt[4]{\frac{4}{3}x^3 + 45}$.

13. D

Differentiate the expression term by term, keeping in mind the product rule and attaching a term $dx$ or $dy$ each time we differentiate $x$ or $y$, respectively. Then solve for $\frac{dy}{dx}$. We get

$$(2x^3y^2) - (5x^2y) = 18$$

$$(4x^3ydy + 6x^2y^2dx) - (10xydx + 5x^2dy) = 0$$

$$4x^3ydy - 5x^2dy = 10xydx - 6x^2y^2dx$$

$$(4x^3y - 5x^2)dy = (10xy - 6x^2y^2)dx$$

$$\frac{dy}{dx} = \frac{(10xy - 6x^2y^2)}{(4x^3y - 5x^2)}.$$

Now evaluate $\frac{dy}{dx}$ at the point (1,2) to get the slope of the tangent line to the curve. We get

$$\left.\frac{dy}{dx}\right|_{(1,2)} = \frac{[10(1)(2) - 6(2)^2(1)^2]}{[4(2)(1)^3 - 5(1)^2]} = -\frac{4}{3}.$$

14. B

Cross-multiply and integrate to get $\int y\ dy = \int 2x^3 dx$. Thus, $\frac{y^2}{2} = \frac{x^4}{2} + C$. Solving for $y$, we see that $y = \pm\sqrt{x^4 + C}$. For example, when $C = 0$, we have $y = \pm x^2$. These integral curves are given in (B).

15. D

To calculate the area between the curves $y = 2x^2 + 1$ and $y = 2x + 5$, we must evaluate the integral $\int_a^b (2x + 5) - (2x^2 + 1)dx$. To determine which values to use for $a$ and $b$ as the limits of the integral, we calculate the $x$ values where the two curves intersect. Solve $2x^2 + 1 = 2x + 5$ by factoring to get $x = 2$ and $x = -1$. Set $a = -1$, $b = 2$. The enclosed area, $A$, is therefore given by the equation

$$A = \int_{-1}^{2} (2x + 5) - (2x^2 + 1)\ dx = \int_{-1}^{2} -2x^2 + 2x + 4\ dx = \left[\frac{-2x^3}{3} + x^2 + 4x\right]_{-1}^{2} = -6 + 15 = 9.$$

16. D

Because $y = 2x^{\frac{3}{2}}$, we get $y' = 3\sqrt{x}$. So the slope of the tangent at $x$ is $3\sqrt{x}$. $2x + 3y = 6$ implies $y = \frac{-2}{3}x + 2$, so the slope of the line given is $\frac{-2}{3}$. Recall that perpendicular lines have negative reciprocal slopes. Thus we must solve the equation $3\sqrt{x} = \frac{3}{2}$ for $x$. We get $x = \frac{1}{4}$. Therefore $\left(\frac{1}{4}, \frac{1}{4}\right)$ is the point where the slope of the tangent is perpendicular to the line $2x + 3y = 6$.

17. A

The equation to use is $V = \int_a^b \pi[f(x)]^2 \; dx$, where $f(x)$ = radius of a cross section given by the function and $a$ and $b$ are the left and right bounds of the surface of revolution. The $\pi[f(x)]^2$ term measures the area of a cross-sectional disk and the $\int_a^b dx$ term adds them up to give the entire volume of the solid. So,

$$V = \int_1^3 \pi[x^{\frac{3}{2}}]^2 \, dx = \int_1^3 \pi x^3 \, dx = \left.\frac{\pi x^4}{4}\right|_1^3 = \pi\left[\frac{3^4}{4} - \frac{1}{4}\right] = 20\pi$$

18. A

To get the equation of the tangent line, we need to know the slope of the tangent line and a point on the tangent line. To calculate the slope of the tangent at $x = \frac{\pi}{8}$, find $y'\left(\frac{\pi}{8}\right)$.

Using the chain rule, $y'\left(\frac{\pi}{8}\right) = 2\cos\left(2x + \frac{3\pi}{4}\right) \cdot 2 = 4\cos\left(2x + \frac{3\pi}{4}\right) = 4\cos\left(\frac{\pi}{4} + \frac{3\pi}{4}\right) = -4$ . Because the line we want is tangent to the curve $y = 2\sin\left(2x + \frac{3\pi}{4}\right)$, $\left(\frac{\pi}{8}, y\left(\frac{\pi}{8}\right)\right) = \left(\frac{\pi}{8}, 0\right)$ is a point on the tangent line. Using the point-slope form we conclude that the equation of the tangent line is $y - \frac{\pi}{2} = -4x$.

19. E

Set $f(x) = \int_0^x \cos(t^2)dt$. Therefore, $f(x^2) = \int_0^{x^2} \cos(t^2)dt$. From the Fundamental Theorem of Calculus we know that $f'(x) = \cos(x^2)$. Now applying the chain rule we see that $\frac{d}{dx}\left(\int_0^{x^2} \cos(t^2)\,dt\right) = \frac{d}{dx} f(x^2) = f'(x^2) \cdot 2x = \cos((x^2)^2) \cdot 2x = 2x\cos(x^4)$.

20. D

This question is a direct application of the intermediate value theorem, which states that if $f$ is a continuous function on the closed interval $[a,b]$ and $d$ is a real number between $f(a)$ and $f(b)$, then there exists a $c$ in $[a,b]$ such that $f(c) = d$. Because $f(0) = 1$, $f(5) = -4$, by the Intermediate Value Theorem there clearly exists a $c_1$ between 0 and 5 such that $f(c_1) = 0$. Similarly, because $f(0) = 1$, $f(-5) = -3$, there exists a $c_2$ between 0 and $-5$ such that $f(c_2) = 0$. Therefore, $c_1$ and $c_2$ are at least two zeros of $f$ and by their location, (D) must be the correct answer.

## FREE-RESPONSE

21. (a) The position of the particle is given by

$x(t) = \int v(t)dt = \int 5t^2 - 4t + 7dt = \frac{5}{3}t^3 - 2t^2 + 7t + C$. Solve for $C$ by substituting $x(3) = 8$: $8 = \frac{5}{3}(27) - 2(9) + 21 + C = 45 - 18 + 21 + C = 48 + C$. Therefore, $C = -40$ and the position of the particle can be written

$$x(t) = \frac{5}{3}t^3 - 2t^2 + 7t - 40.$$

(b) Total distance traveled is given by $\int_0^2 |v(t)|\,dt$.

$v(t)$ is positive everywhere, so this is just

$$x(2) - x(0) = \left(\frac{5}{3}2^3 - 2 \cdot 2^2 + 7 \cdot 2 - 40\right) - \left(\frac{5}{3}0^3 - 2 \cdot 0^2 + 7 \cdot 0 - 40\right) = \frac{40}{3} - 8 + 14 = \frac{58}{3}.$$

(c) We find the extrema by setting $v'(t) = 0$. $v'(t) = 10t - 4 = 0$ when $t = \frac{2}{5}$. Note that $v''(t) = 10 > 0$, so this value is a minimum.

The position is, then, $x\left(\frac{2}{5}\right) = \frac{5}{3}\left(\frac{8}{125}\right) - 2\left(\frac{4}{25}\right) + \frac{14}{5} - 40 = -37\frac{31}{75}$.

Note: Any question that asks you to compute total distance is asking for $\int |v(s)|\,ds$, where $v$ is the velocity. Our problem is easier, since $|v(s)| = v(s)$ for our function. For part (c), it isn't enough to just compute the zero of $v'$. You also need to explain, however briefly, why $v$ actually attains a minimum (and doesn't just continue to decrease to $-\infty$ ). In the solution, this is because we stated that $v$ is positive everywhere, and so can't go to $-\infty$.

# DIAGNOSTIC TEST AB: CORRELATION CHART

Use the results of your test to determine which topics you should spend the most time reviewing.

| Question Number | Topic |
|---|---|
| 1 | Definition of Derivative |
| 2 | Integration of Trigonometric Functions |
| 3 | Relationship Between Concavity of $f$ and the Sign of the Second Derivative |
| 4 | Quotient Rule, Chain Rule, and Logarithmic Derivatives |
| 5 | Relationship Between Increasing/Decreasing Property of $f$ and the First Derivative |
| 6 | Chain Rule and Trigonometric Derivatives |
| 7 | Estimating Limits of a Function from the Graph |
| 8 | Trigonometric Limits |
| 9 | Graphical Relationship Between Increasing/Decreasing Nature of $f$ and the Sign of $f'$ |
| 10 | Evaluation of Limits As $x$ Approaches Infinity |
| 11 | Evaluating Indeterminate Limits Using Algebraic Techniques |
| 12 | Solving Differential Equations with the Initial Condition |
| 13 | Implicit Differentiation |
| 14 | Graphical Interpretation of Solutions to Differential Equations |
| 15 | Integration Within Specified Boundaries |
| 16 | Derivatives As Slope of Tangent |
| 17 | Integration, Volume of a Solid with Known Cross-Section |
| 18 | Derivatives, Chain Rule, Derivatives As Slope of Tangent |
| 19 | Fundamental Theorem of Calculus and Chain Rule |
| 20 | Intermediate Value Theorem |
| 21 | (a) Integration Using Initial Value Data<br>(b) Integration to Determine Total Displacement<br>(c) Differentiation to Find Extremum Value |